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A short appraisal of the partitioning of FM signals

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Summary

This report shows how a simple mathematical equation can conveniently be used to gain an understanding of an FM-signal partitioning technique. The bandwidth requirements associated with partitioning are then examined; it is found that a total bandwidth greater than that required by the original signal is necessary. Some considerations of signal-to-noise ratio follow, and the possibility of recovering the output signal from one or more of the partitioned signals is explored.

1. Introduction

Frequency multiplication has for a long time been recognised as a method of increasing the deviation of a frequency modulated signal. Two systems have been proposed[1,2] which realise, in effect, the inverse process in which an FM signal of given deviation is partitioned into a number of lower-deviation signals that can be handled by separate low-bandwidth transmission channels. The technique could be useful, for example, in video recording; it has in fact been successfully used to permit a 12 MHz carrier, frequency modulated by a 10 MHz television signal, to be recorded using two tracks carrying lower-bandwidth signals.[1]

This report presents a simplified theoretical approach to such systems and makes some observations about bandwidth and signal-to-noise ratio.

2. The partitioning technique

It is convenient to approach the principle of FM signal partitioning by first considering the following mathematical relationship.[3]

$$\text{Sin}\, A = 2^{N-1}\, \text{Sin}\left[\frac{A}{N}\right] \text{Sin}\left[\frac{A+\pi}{N}\right] \text{Sin}\left[\frac{A+2\pi}{N}\right] \cdots\cdots \text{Sin}\left[\frac{A+(N-1)\pi}{N}\right] \qquad (1)$$

where N is any integer.

This equation reveals that any sine wave may be expressed as the product of a number of sub-harmonics having equal frequencies but different phases.*

* There are other possible variations — for example:

$$\text{Cos}\, A = 2^{N-1}\left[\text{Cos}\frac{A}{N} - \text{Cos}\frac{\pi}{2N}\right]\left[\text{Cos}\frac{A}{N} - \text{Cos}\frac{3\pi}{2N}\right] \cdots\cdots \left[\text{Cos}\frac{A}{N} - \text{Cos}\frac{(2N-1)\pi}{2N}\right]$$

i.e. a product series of sub-harmonics having equal frequencies but each added to a different d.c. term. Such variations lead to similar instrumentation, however.

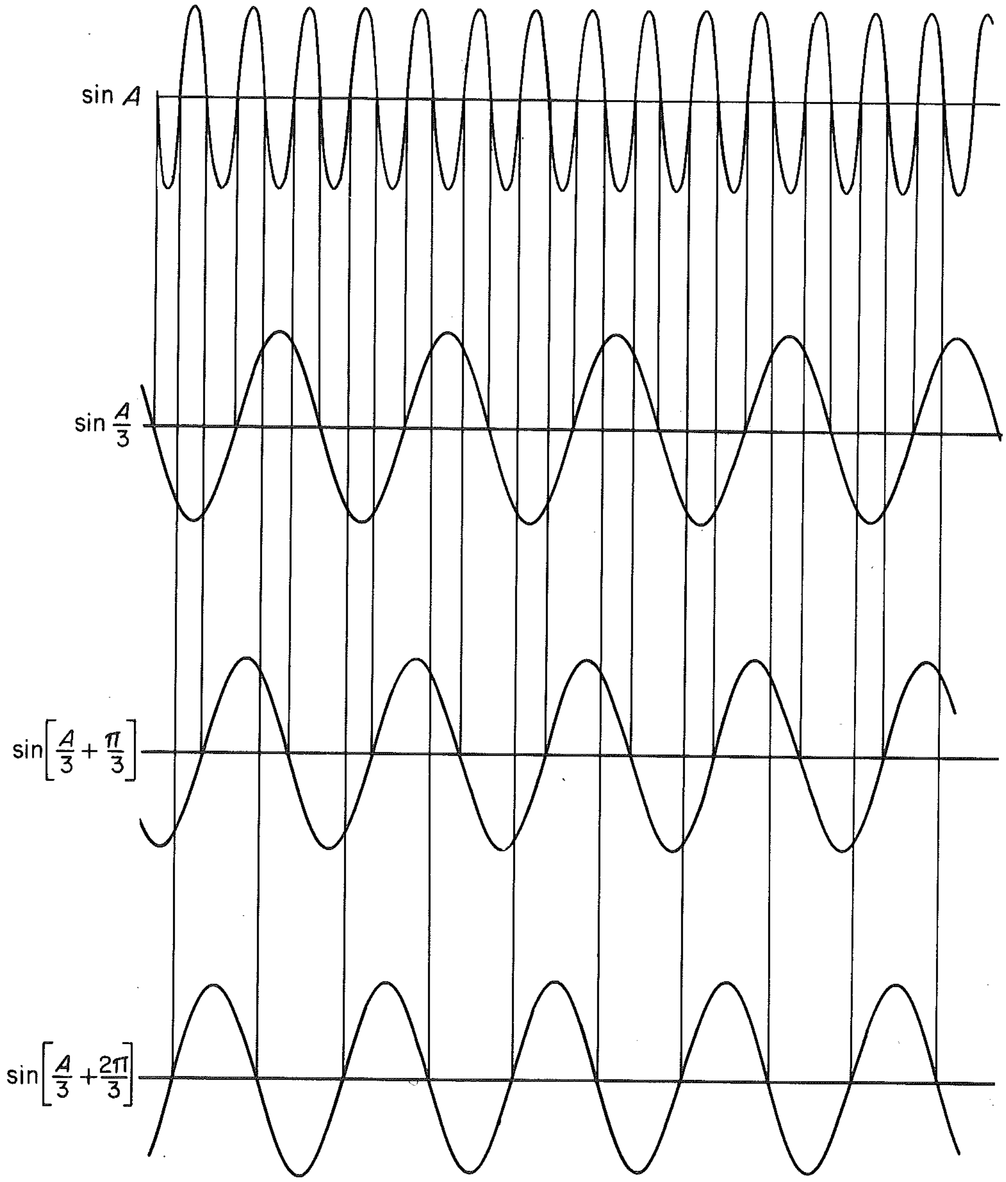

Fig. 1 - Simple example illustrating use of product formula

This relationship is illustrated in Fig. 1 which is drawn for the example N = 3. The top curve, Sin A, may be obtained by multiplying the other three together, the normalising factor in this case being $2^{(3-1)} = 4$.

The formula continues to apply if Sin A corresponds to an angle-modulated wave, i.e. if it becomes

$$f(t) = \text{Sin } [\omega_c t + \tau(t)]$$

Fig. 2 illustrates four applications of the formula to a frequency modulated wave, for the cases where N = 2, 3, 4 and 5. The original curve may again be obtained by multiplying the sub-harmonic components together.

The practical application of the product formula to the partitioning of FM signals is based on the fact that all the useful information contained in an FM waveform is carried by the positions of its zero-crossing points.

As Sin A goes through its zero-crossing points, zero-crossing points occur in one or other of the sub-harmonic terms. This fact is emphasised in Figs. 1 and 2 by the lines linking the crossing points of the top curves to those of the other curves. Thus crossing points 1, (N + 1), (2N + 1), (3N + 1), etc., for example, of Sin A correspond to the crossing points of Sin A/N, while crossing points 2, (N + 2), (2N + 2), (3N + 2) etc. of Sin A correspond to the crossing points of Sin$[A + (N - 1)\pi]$/N. As Fig. 2 shows, this state of affairs continues when the positions of zero-crossing points are disturbed by frequency modulation. Thus if Sin A is a frequency modulated wave, the essential information within it may be considered to be equally divided between the sub-harmonic components, these components making their contribution by supplying crossing-point information in cyclic order.

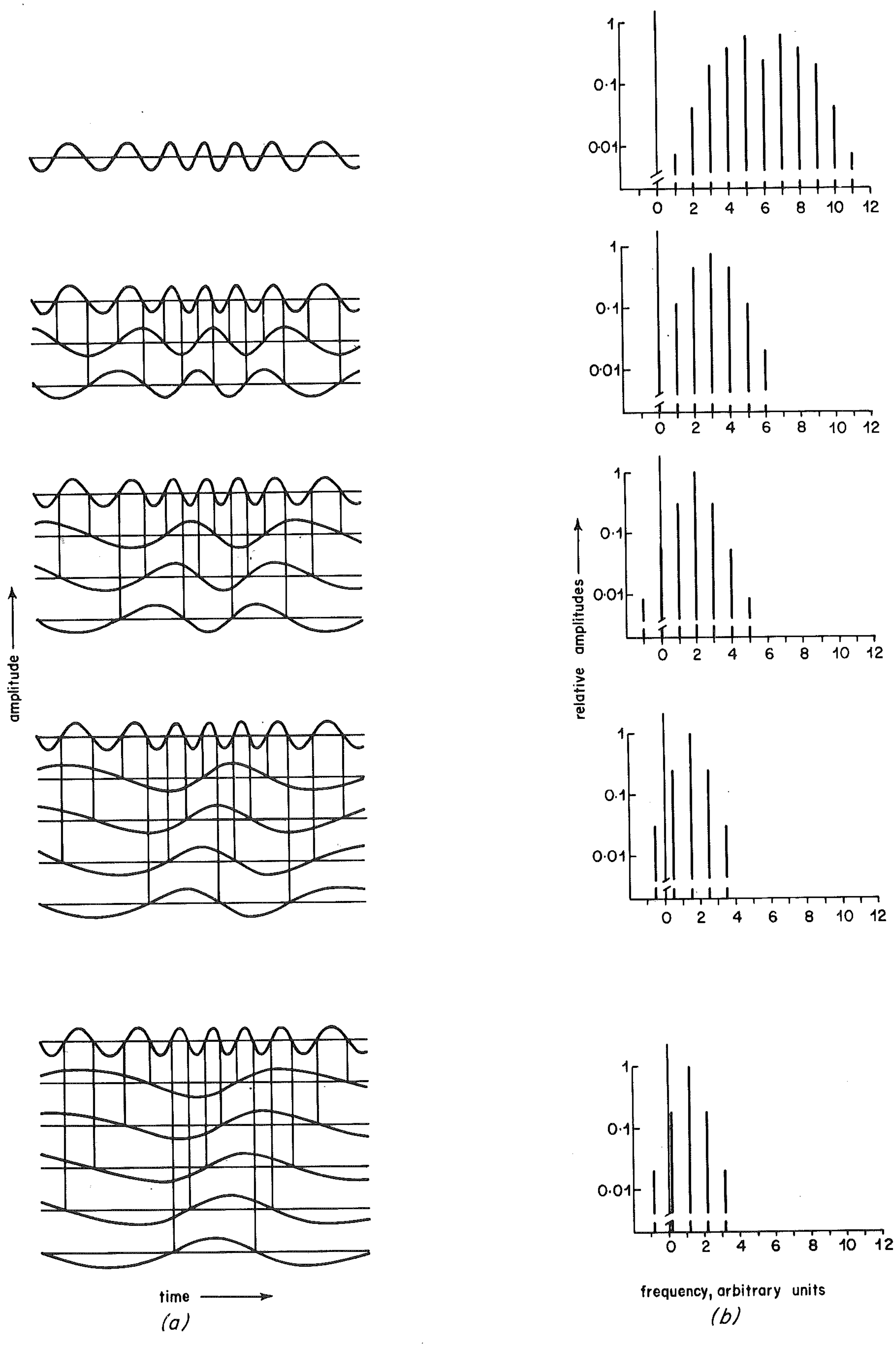

Fig. 2 - A frequency modulated wave partitioned into 2, 3, 4 and 5 components showing J() factors determining corresponding spectra

(*a*) Original wave and partitioned components (*b*) Spectrum components

The proposed systems[1,2] make use of these facts by arranging that frequency-modulated square waves, having an unmodulated frequency equal to 1/N times the carrier frequency of the original frequency-modulated wave, are propagated through each of N low-bandwidth channels.* The transitions ('zero-crossing points') of these N square waves are generated in turn at times when the original waves goes through its crossing points. The original wave is then reconstructed at the receiving terminal by causing a bi-stable circuit to change state whenever a transition arrives from any one of the low-bandwidth channels, and then low-pass filtering the output of the bi-stable circuits to remove the harmonics. This adding together of transitions is equivalent, so far as crossing points are concerned, to the multiplication of sub-harmonic terms shown in Equation (1).

3. Bandwidth considerations

3.1. Sinusoidal signals

If Sin A takes the form $f(t) = \text{Sin}[\omega_c t + \beta \,\text{Sin}\, \omega_m t]$, i.e. if the wave is frequency modulated with modulation index β, Equation (1) becomes:

$$f(t) = 2^{N-1}\,\text{Sin}\left[\frac{\omega_c}{N}t + \frac{\beta}{N}\,\text{Sin}\,\omega_m t\right]\,\text{Sin}\left[\frac{\omega_c}{N}t + \frac{\beta}{N}\,\text{Sin}\,\omega_m t + \frac{\pi}{N}\right]\ldots$$

$$\ldots.\text{Sin}\left[\frac{\omega_c}{N}t + \frac{\beta}{N}\,\text{Sin}\,\omega_m t + \frac{(N-1)\pi}{N}\right] \qquad (2)$$

$$= 2^{N-1}\prod_{n=0}^{n=(N-1)}\text{Sin}\left[\frac{\omega_c}{N}t + \frac{\beta}{N}\,\text{Sin}\,\omega_m t + \frac{n\pi}{N}\right]$$

Now the instantaneous angular frequency of each of the sub-harmonic terms may be obtained by differentiating

$\left[\frac{\omega_c}{N}t + \frac{\beta}{N}\,\text{Sin}\,\omega_m t + \frac{n\pi}{N}\right]$ with respect to t to give

$$\left[\frac{\omega_c}{N} + \frac{\beta\omega_m}{N}\,\text{Cos}\,\omega_m t\right]$$

The instantaneous angular frequency of the composite wave, on the other hand, is

$$\frac{d}{dt}\left[\omega_c t + \beta\,\text{Sin}\,\omega_m t\right]$$

$$= \left[\omega_c + \beta\omega_m\,\text{Cos}\,\omega_m t\right]$$

* Square waves are chosen because they permit information about zero-crossing points to be conveniently conveyed using bi-stable circuits. In the low bandwidth channels, the waves lose their square form, but this does not matter so long as the crossing points are not significantly shifted.

Therefore the peak frequency deviation $\pm(\beta\omega_m)/(2\pi N)$, of each of the sub-harmonic terms is (1/N) times that of the composite wave, but the modulating frequency, $(\omega_m)/(2\pi)$, associated with the sub-harmonics is equal to that applied to the composite wave. As Fig. 2 illustrates, therefore, the bandwidth of each of the sub-harmonic signals is greater than (1/N) of that of the original signal.

This means that the sum of the bandwidths required for the sub-harmonic channels is greater than that required for the composite signal. Assuming double-sideband modulation and that Carson's Law (FM signal bandwidth = 2 max. frequency deviation + 2 max. modulating frequency) applies, the total bandwidth required by the sub-harmonics channel is equal to

$$\sum b = N.2\left[\frac{\beta\omega_m}{2\pi N} + \frac{\omega_m}{2\pi}\right]$$

while that occupied by the composite signal is

$$B = 2\,.\left[\frac{\beta\omega_m}{2\pi} + \frac{\omega_m}{2\pi}\right]$$

The excess bandwidth required by the partitioned signal, expressed as a fraction of that required by the composite signal is then

$$\frac{\sum b - B}{B} = \frac{\omega_m(N-1)}{\beta\omega_m + \omega_m}$$

$$= \frac{N-1}{\beta+1}$$

A further feature of FM partitioning should be taken into account, however. Since the carrier frequency is reduced by partitioning but the modulating frequency is not, there must be a value of N for which sidebands of significant magnitude on the lower side of the partitioned carrier extend 'beyond' dc; i.e. the spectrum of the partitioned signal is folded about d.c. Clearly, if N is sufficiently large, both the frequency and deviation of the partitioned carrier will approach zero, the bandwidth will be primarily determined by the modulating signal, and all the lower sidebands will be folded. Now folded sidebands are normally not permitted in FM; they produce interfering beat patterns that prevent the original modulating function from being recovered. This restriction need not be applied to FM partitioning, however, in which information about

successive crossing points is correctly conveyed to the receiving terminal regardless of the number of channels used. In this respect, FM partitioning has advantages comparable to those of polyphase a.m.[4]

This property is also illustrated in Fig. 2. In three of the examples shown, sub-harmonics are produced which have negative frequency components. Nevertheless the crossing points of the original wave continue to be correctly conveyed by the sub-harmonics.

When negative frequency components are present, the bandwidth required for each of the partitioned signals is of course less than the above expression would suggest, being instead:

$$b_1 = \left[\frac{\beta\omega_m}{2\pi N} + \frac{\omega_m}{2\pi} + \frac{\omega_c}{2\pi N}\right]$$

3.2. Square-wave signals

The above discussion is relevant to the partitioning in terms of the generation of frequency modulated sinusoidal signals such as those shown in Fig. 2. It should not always be assumed, however, that the bandwidths prescribed above for partitioned signals are sufficient in the case when the latter are derived by means of binary circuits operating on zero-crossing points. The reason for this is that limiting an FM signal creates a whole series of spectral components centred on odd multiples of the carrier frequency. If the original spectrum contains significant components the ratio of whose frequencies can exceed 3 : 1, some of the components introduced as sidebands of 3 times the carrier will intrude into the original frequency band. It is then not possible to recover the original frequency modulated signal with precision by simple low-pass filtering. The squaring process will, by definition, preserve the positions of the original crossing points, but the subsequent filtering disturbs them.

Now if the original signal to be partitioned is squared, new components may or may not be introduced into its spectral band. If N is large enough, however, the partitioned squared signals will certainly have aliased components within their basebands. When this is the case, the bandwidth necessary to convey the partitioned signals without significant disturbance to the crossing points may be greater than the above arguments, which assumed the use of sinusoidal signals throughout, have suggested. The implications of this statement would have to be worked out for each envisaged application, taking into account the parameters proposed and the degree of crossing-point disturbance that could be tolerated. The experimental system described in Ref. 1, for example, would appear to be vulnerable to errors introduced by low-pass filtering, but has in practice given acceptable results.

4. Signal-to-noise ratio considerations

Provided that the signals in individual channels do not contain negative frequency components, and that their crossing points have not been disturbed by excessive low-pass filtering, the original modulating wave may be obtained from any one of them by direct demodulation. If folded sidebands are present, it would be possible to remove them by combining a sufficient number ($\leqslant N$) of the signals before demodulating.

Now if the individual channels introduce noise, the effect of this noise on the signal obtained by demodulating the composite output is reduced as the number of channels contributing to the output is increased. As the number of contributing channels is increased, the composite deviation, and hence the demodulated signal amplitude, is increased linearly; the output noise power is also increased linearly and the output noise-voltage therefore increases according to a half-power law. Thus if a given signal-to-noise ratio is required for the composite FM signal, then a signal-to-noise ratio 10 log N dB less than this is sufficient on each of the low-bandwidth channels. Where the low-bandwidth channels have a performance better than this they need not all be used.

One outcome of this is that if a number of low-bandwidth channels are grouped into sets, each set carrying one programme signal, and the channels are subject to differing fluctuations in noise level, the number of channels allocated to each set may be varied in such a way as to equalise the output signal-to-noise ratios.

5. Conclusions

FM signal partitioning may be readily understood in terms of a product expansion. It has been shown that, in general, the total bandwidth occupied by the N channels into which the original signal is divided is greater than that required for the original signal. The total bandwidth actually required depends on a number of complicated factors, not the least being the degree of distortion and intermodulation products permitted in the final signal. The modulating signal may in general be recovered by using only some, rather than all, of the N channels, this saving being made in exchange for a reduction in output signal-to-noise ratio.

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